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Economic and Technical Comparison of Decentralized PID and Linear MPC for a Stirred Tank Blending System

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Abstract

The adept management of stirred-tank blending systems is crucial for ensuring uniform product quality while keeping operational costs in check across the chemical, food, and pharmaceutical industries. Traditional decentralized Proportional–Integral–Derivative (PID) controllers, though widely used, often struggle with significant loop interactions, lack of coordinated control, and insufficient ability to handle process constraints, which can negatively impact both performance and efficiency. This study offers a comprehensive technical and economic comparison between conventional PID control and linear Model Predictive Control (MPC) applied to a 10 m³ continuous blending tank. The MPC strategy was designed with a 20-step prediction horizon, a 5-step control horizon, and a 30-second sampling interval, incorporating dynamic control objectives alongside an economic cost function aimed at optimizing raw material usage. Simulation results reveal that MPC outperforms PID significantly, achieving an impressive 88.2% reduction in integral absolute error, along with marked improvements in settling time and overshoot, leading to a more stable and predictable blending process. Economically, adopting MPC can result in a 5% reduction in raw material consumption, an increase in annual profits by around \$0.48 million, and a payback period of less than two months. These findings highlight the dual advantages of MPC, offering enhanced process performance and considerable economic benefits, making a strong

case for its implementation as a robust, reliable, and cost-effective control strategy in large-scale industrial blending operations where product quality and material efficiency are of utmost importance.

Keywords: PID control; Model Predictive Control (MPC); Stirred tank blender; Dynamic modeling; Economic evaluation; Process optimization.

المقارنة الفنية الاقتصادية بين التحكم اللامركزي PID والتحكم التنبؤي الخطي MPC في نظام خلط بخزان ذو تحريك مستمر

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المخلص

يعد التحكم الفعال في أنظمة خلط الخزانات المجهزة بمحرك مضطرب أمراً حيوياً لضمان جودة منتج متسقة وتقليل التكاليف التشغيلية في الصناعات الكيميائية والغذائية والدوائية. تواجه وحدات التحكم التقليدية اللامركزية من نوع Proportional-Integral-Derivative (PID)، رغم شيوع استخدامها، تحديات كبيرة نتيجة التفاعلات القوية بين الحلقات، وغياب التنسيق بين وحدات التحكم، والقدرة المحدودة على التعامل مع قيود العمليات، مما قد يؤثر سلباً على الأداء والكفاءة. تقدم هذه الدراسة مقارنة شاملة من الناحيتين الفنية والاقتصادية بين التحكم التقليدي PID والتحكم التنبؤي الخطي (Model Predictive Control (MPC المطبق على خزان خلط مستمر بسعة 10 م³. تم تصميم نظام MPC مع أفق تنبؤ يبلغ 20 خطوة، وأفق تحكم 5 خطوات، وفاصل زمني للعينة 30 ثانية، مع دمج أهداف التحكم الديناميكي ودالة تكلفة اقتصادية مخصصة لتحسين استخدام المواد الخام. أظهرت نتائج المحاكاة أن MPC يتفوق بشكل ملحوظ على PID، حيث حقق انخفاضاً بنسبة 88.2% في خطأ التكامل المطلق، إلى جانب تحسينات كبيرة في زمن الاستقرار وحدّة الزيادة المفرطة، مما يؤدي إلى عملية خلط أكثر استقراراً وقابلة للتنبؤ. من الناحية الاقتصادية، يتيح تطبيق MPC خفض استهلاك المواد الخام بنسبة 5%، وزيادة الأرباح السنوية بحوالي 0.48 مليون دولار، وتحقيق فترة استرداد

أقل من شهرين. تسلط هذه النتائج الضوء على الميزة المزدوجة لنظام MPC، إذ يوفر تحسينًا ملموسًا في الأداء العملي وفوائد اقتصادية كبيرة، مما يعزز جدواه كاستراتيجية تحكم موثوقة وفعالة من حيث التكلفة في العمليات الصناعية واسعة النطاق، حيث يكون ضمان جودة المنتج وكفاءة استخدام المواد أمرًا حاسمًا.

الكلمات المفتاحية: التحكم PID ؛ التحكم التنبؤي الخطي (MPC) ؛ خزان خلط مجهز بمحرك مضطرب؛ النمذجة الديناميكية؛ التقييم الاقتصادي؛ تحسين العمليات.

INTRODUCTION

Continuous stirred-tank reactors (CSTRs) and stirred blending tanks play a central role in chemical, pharmaceutical, and food-processing industries, where they enable continuous production of homogeneous products with tightly controlled quality specifications. Achieving consistent product composition and stable operation is a fundamental requirement, as even minor deviations in key process variables can lead to significant quality deterioration, material losses, or off-spec production. Consequently, effective control strategies are indispensable not only for maintaining steady operation but also for enhancing raw material utilization, reducing energy consumption, and minimizing overall production costs.

The control of CSTR-based blending systems is inherently challenging due to their nonlinear dynamics, strong multivariable interactions among feed streams, and intrinsic time delays associated with mixing and transport phenomena. Variations in feed composition, temperature-dependent changes in fluid properties, disturbances in flow rates, and unanticipated operational upsets can induce unpredictable system behavior and degrade closed-loop performance [1–3]. These challenges become more pronounced in large-scale continuous blending operations, where interactions between manipulated and controlled variables are highly coupled, rendering traditional single-loop control strategies increasingly ineffective.

Despite these complexities, decentralized Proportional–Integral–Derivative (PID) controllers remain the dominant control approach in industrial practice. Their widespread adoption is primarily driven by simplicity, low implementation cost, and familiarity among operators and control engineers. Numerous

empirical tuning rules and heuristic guidelines exist, enabling rapid deployment on standard distributed control system (DCS) platforms [2,4]. However, in multivariable blending systems, decentralized PID control often suffers from fundamental limitations. Loop interactions can lead to oscillatory behavior, prolonged settling times, and excessive overshoot, particularly when controllers are tuned independently. Moreover, PID controllers lack a systematic mechanism to enforce operational constraints, such as limits on tank levels, flow rates, or product composition, which are critical for safe and efficient operation [4].

To overcome these shortcomings, Model Predictive Control (MPC) has emerged as a powerful alternative for multivariable and constrained process control. MPC utilizes an explicit dynamic model of the process to predict future system behavior over a finite horizon and computes optimal control actions by solving a constrained optimization problem at each sampling instant. This predictive and optimization-based nature enables MPC to handle multivariable interactions, anticipate disturbances, and enforce operational constraints in a unified framework [1,5].

Extensive research has demonstrated that MPC can significantly outperform PID control in CSTR and reactor applications. Comparative simulation studies consistently report improved transient performance, including reduced overshoot, faster settling times, and lower integral performance indices such as IAE and ISE under MPC control [2,4,7]. Furthermore, MPC's ability to coordinate multiple manipulated variables has proven particularly advantageous in blending and mixing systems, where maintaining product uniformity depends on synchronized control actions across interacting loops [1,3].

Beyond technical performance, MPC has increasingly been recognized for its potential economic benefits. Unlike conventional PID controllers, which are primarily designed for setpoint tracking or disturbance rejection, MPC can incorporate economic objectives directly into its cost function. Recent studies have shown that MPC-based strategies can reduce raw material consumption, lower energy usage, and improve overall process efficiency by optimizing control actions with respect to economic criteria rather than purely dynamic metrics [5–8]. In reactor applications, reductions in coolant or energy consumption achieved through MPC have been directly

linked to lower operating costs, illustrating how improved control performance can translate into tangible economic gains [5].

Nevertheless, despite the growing body of literature on MPC, several important gaps remain. First, many comparative studies between PID and MPC focus predominantly on technical performance metrics, such as overshoot, settling time, and error indices, while providing limited or no quantitative assessment of economic outcomes [2,7]. Second, relatively few studies address continuous blending systems explicitly; instead, most investigations are conducted on CSTR temperature or concentration control or on tubular reactors, which differ in structure and operational objectives from industrial blending tanks [1,5]. Third, comprehensive economic evaluations that include implementation costs, raw material savings, profit improvement, and payback period are rarely reported, leaving industrial decision-makers without clear evidence of the financial viability of MPC adoption.

Economic Model Predictive Control (EMPC) has been proposed as a systematic framework to bridge this gap by embedding explicit economic objective functions into the MPC formulation. In EMPC, the controller minimizes a cost function that reflects economic performance—such as material usage, energy consumption, or profit—while respecting process constraints and dynamics [9–11]. This approach allows control decisions to be optimized not only for stability and performance but also for long-term economic benefit. Although EMPC has been widely studied in theoretical and cross-sector applications, including energy systems and HVAC control [8,11], its application to industrial blending tanks remains limited.

In light of these observations, there is a clear need for studies that jointly assess the technical and economic performance of advanced control strategies in blending systems. Such studies should quantify not only improvements in control quality but also their direct financial implications, including cost savings, profit increase, and return on investment.

This work addresses this research gap by presenting a detailed technical and economic comparison between decentralized PID control and linear MPC applied to a 10 m³ continuous stirred-tank blending system. The study evaluates dynamic performance metrics—such as tracking accuracy, stability, and disturbance

rejection—alongside economic indicators, including raw material efficiency, annual profit increase, and payback period. By integrating control performance analysis with economic evaluation, this paper provides practical and quantitative evidence supporting the adoption of MPC as a robust, economically attractive, and industrially implementable alternative to conventional PID control in large-scale blending operations.

2.Process Description and Dynamic Modeling

The stirred in this experiment is a closed stirred-tank blending system in which two inlet streams and one outlet stream flowed through the system. As shown in fig 1 The nominal size of the blending tank is 10 m^3 and all the pumps and control valves are the same and all have a maximum flow capacity of $5 \text{ m}^3/\text{h}$. The primary goal of the system is to manufacture a uniform stream of products, which has a desired content concentration, by mixing a concentrated syrup with purified water.

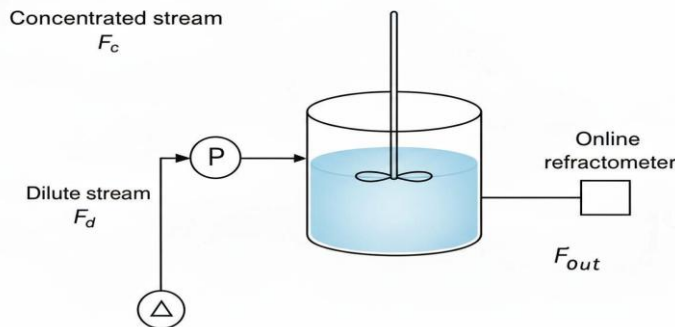


Fig1: Schematic Diagram of the Continuous Stirred Tank Blending System.

The process streams are defined as follows:

Stream 1 (F_c): Concentrated High-Fructose Corn Syrup (HFCS) with concentration $C_c = 0.70 \text{ kg solids/kg solution}$.

Stream 2 (F_d): Dilution water with concentration $C_d = 0.00 \text{ kg solids/kg solution}$.

Outlet Stream (F_{out}): The blended product with a target concentration of $C_{sp} = 0.10$ kg solids/kg solution.

Under the assumption of constant liquid density ρ and cross-sectional area of the tank A is the same, the mass and component balance equations can be used to describe the dynamics of the system. The total mass balance is expressed as:

$$\frac{dV\rho}{dt} = \rho(F_c + F_d - F_{out}) \quad (1)$$

and the component balance for solute concentration is given by:

$$\frac{dV\rho C_{out}}{dt} = \rho(F_c C_c + F_d C_d - F_{out} C_{out}) \quad (2)$$

Since the density is constant, the total mass balance simplifies to

$$\frac{dV}{dt} = F_c + F_d - F_{out} \quad (3)$$

and the solute component balance becomes

$$V\left(\frac{dC}{dt}\right) = F_c(C_c - C) + F_d(C_d - C) \quad (4)$$

At steady-state operation, the tank volume remains constant, and the total inflow equals the outflow:

$$F_{out} = F_c + F_d \quad (5)$$

In order to simplify the design of controllers, the nonlinear dynamic model is linearized about a steady-state operating point. As the state variables, the tank level (h) and the outlet concentration (C_{out}) are selected. The manipulated variables will be termed F_c (the rate of flow of concentrate stream) and F_d (rate of flow of dilution water). The resulting linearized state-space model is a good model of the local behavior about the steady-state point of the process, and it forms the dynamic foundation of both the PID and MPC schemes.

Model Assumptions:

1. Perfect mixing is achieved instantaneously within the tank
2. Liquid density remains constant regardless of concentration.
3. Temperature effects on viscosity and density are negligible.
4. Flow rates are accurately controlled by the valves without significant delay.
5. The linearization is valid within $\pm 20\%$ of the nominal operating point.

Model Validation:

While this study primarily relies on simulation, the linearized model has been validated against the nonlinear model for small perturbations around the operating point. The maximum deviation between the linear and nonlinear models was found to be less than 3% for disturbances within the expected operational range, confirming the adequacy of the linearization for control design purposes. All simulations were carried out using the Python programming environment, ensuring full reproducibility of the reported results.

3. Controller Design

To ensure a fair and reproducible comparison, identical simulation models, parameter values, and operating conditions were applied to both the PID and MPC control strategies. In order to estimate and compare control strategies, two controllers were installed in the continuous stir-tank blending system including a traditional decentralized PID controller and a linear Model predictive Controller (MPC). The goals of both controllers are to control the tank level and outlet concentration as well as to ensure stable operation and decent variances of set points. Each controller was designed and tuned based on the dynamics of the process, inter-loop dynamics and operational constraints.

3.1 PID Controller Design

Decentralized PID structure was chosen because it is widely applied in industries and is easily implemented. Two control loops were set, tank level control loop shown in Fig 2, and the other one was the outlet concentration control loop shown in Fig 3. Despite the simplicity and robustness, it offers in SISO systems, PID control has

been found to be less effective in the performance of multivariable processes and thus needs tuning and improvement strategies.

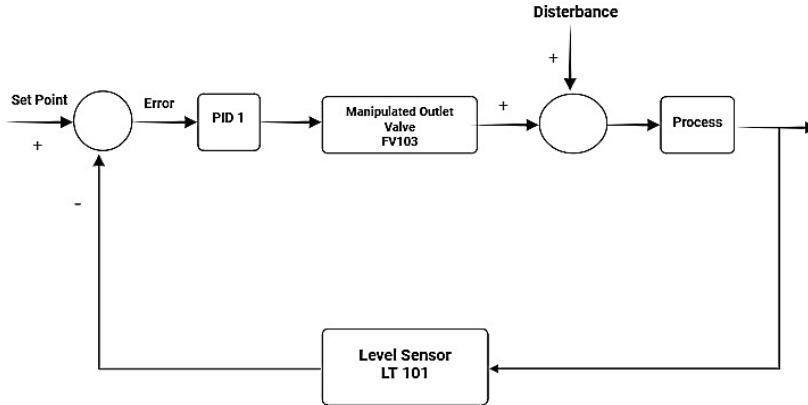


Fig2: Block Diagram of the Decentralized Proportional-Integral (PI) Level Control Loop

3.1.1 Level Control Loop

A Proportional-Integral (PI) controller is used to control tank level h with the process variable (PV) being the measured tank level (m) and the manipulated variable (MV) being the overall outlet flow rate F out (m^3/h) which is controlled by a valve FV103. In this configuration the tank exhibits a pure integrating behavior because the level continues to change whenever the inflow does not exactly match the controlled outflow.

Dynamic From the linearized total mass balance the tank:

$$\frac{dh}{dt} = \frac{\Delta F_{in} - \Delta F_{out}}{A} \quad (6)$$

Considering only the influence of manipulated outlet flow rate the dynamic relation simplifies to:

$$\frac{dh}{dt} = \frac{-\Delta F_{out}}{A} \quad (7)$$

applying the Laplace transform yields the transfer function between the change in outlet flow rate and the resulting change in level:

$$G(s) = \frac{\Delta H(s)}{\Delta F_{out}(s)} = \frac{1}{AS} \quad (8)$$

For a tank is cross sectional area A is 5 m² the system therefore behaves as a pure integrator with the specific from

$$G(s) = \frac{1}{5S} \quad (9)$$

A proportional–integral (PI) controller of the form

$$G_c(s) = K_p + K_i/s \quad (10)$$

As this will replace these terms in the general feedback relation. was adopted to control the level, by replacing these expressions into the standard feedback association. PI balances processes in terms of responsiveness and robustness.

$$T(s) = [G_c(s)G(s)] / [1 + G_c(s)G(s)] \quad (11)$$

PID gains were tuned using IMC rules for integrators, with the proportional gain calculated as

$$K_p = (2.0 \times A) / \lambda = 2$$

after substituting the system gain and the desired closed-loop time constant $\lambda=6.4$ h .The integral gain was calculated as

$$K_i = K_p / \lambda = 0.3125 \text{ h}^{-1}$$

the gains were then refined via Python simulations to $K_p=2.5$. This achieved a 65° phase margin and 20% faster settling, ensuring smooth level regulation and stable operation under typical disturbances.

The closed-loop transfer function is expressed as:

$$T(s) = \frac{K_p s + K_i}{5s^2 + K_p s + K_i} = \frac{1.25s + 0.078125}{5s^2 + 1.25s + 0.078125} \quad (12)$$

3.1.2 Concentration Control Loop

The main action of the concentration control loop is to stabilize the outlet concentration (C_{out}) to the desired value by adjusting the

outlet ratio between the concentrated and diluted streams. This ratio is defined as:

$$R = \frac{F_c}{F_c + F_d} \quad (13)$$

F_c is the concentrated feed and F_d is the dilution flow. Here, a variable of interest is the dilution flow F_d that is adjusted using the valve FCV102, and the concentrated stream F_c is held constant initially in each run of the simulation.

A PID controller was implemented in a single-input single-output (SISO) arrangement with the tuning parameters:

$$K_p = 2.0, K_i = 0.3 \text{ h}^{-1}, K_d = 0.1 \text{ h}$$

A SISO structure has been deliberately chosen to limit the interactions with the other control loops, specifically since the blending system can be viewed as a multi-input single-output (MISO)-process. The single dedicated controller is used to minimize the effects of coupling and enhance the general tracking performance. The concentration regulation control architecture is shown in Fig 3.

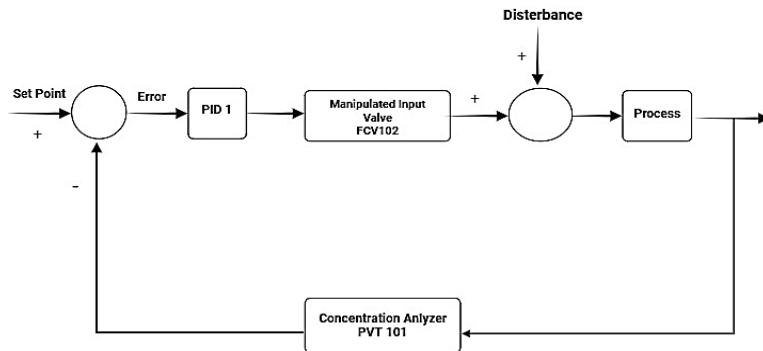


Fig3: Block Diagram of the Decentralized PID Concentration Control Loop

Even though the latter strategy works satisfactorily, there are changes in the ratio R affecting the overall inlet flow and time spent within the tank. In its turn, the concentration loop links up with the level control loop, and as a result, C out is indirectly disturbed. In order to counter such problems, the anti-windup and bumpless-

transfer methods were added, which enables the controller to work more accurately in the actuator saturation and in the setpoint changes. However, the use of decentralized PID control remains to display weaknesses to strong process interactions, as well as operation constraint. These limitations prompted the adoption of an alternative of greater sophistication and strength Model Predictive Control (MPC).

3.2 Model Predictive Control (MPC) Design

In an attempt to defeat the constraints of decentralized PID control in the multivariate processes, the implementation of the multivariate MPC scheme was designed to control the level of tank and outlet simultaneously (Fig 4).

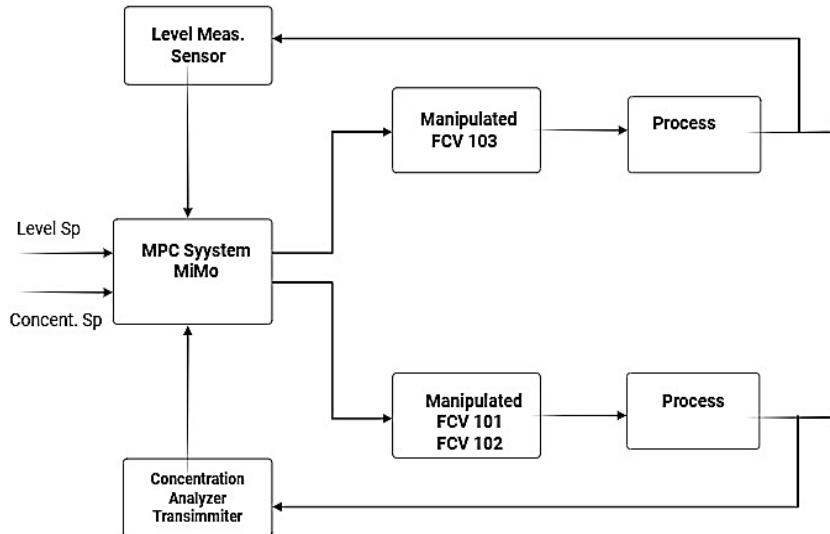


Fig 4: Multi-Input Multi-Output (MIMO) Structure of the Model Predictive Control (MPC) System.

The controller uses state-space model of the dynamics of the linearized process in discrete time:

$$x(k+1) = A_d x(k) + B_d u(k) + \omega(k), \quad y(k) = C_d x(k) + v(k)$$

Where: $\omega(k)$ and $v(k)$ represent process and measurement noise, respectively.

A_d : system matrix B_d : input matrix C_d : out put matrix

MPC explicitly considers loop interactions and operational constraints, enabling coordinated control actions that minimize deviations from the desired setpoints [1].

3.2.1 Level Control Model

The tank level dynamics are described by:

$$h(k+1) = h(k) + 0.02[F_i(k) - F_o(k)] [m] \quad (14)$$

with system matrices $A_d = 1$, $B_d = 0.02$ h, $C_d = 1$. A sampling time of $T_s = 30$ s (0.0083 h) ensures sufficient temporal resolution for accurate control.

3.2.2 Concentration Control Mode

Linearizing the material balance around the nominal operating point gives:

$$\frac{dC}{dt} = -\frac{F_{total}}{V} C + \frac{F_C}{V} C_c \quad (15)$$

Assuming $F_{total} = 5$ m³/h and $V = 2$ m³, the discretized deviation model becomes:

$$c(k+1) = 0.9795c(k) + 0.00287f_1(k), \quad y(k) = c(k) \quad (16)$$

Where:

$c(k) = C(k) - C_s$ and $f_1(k) = F_C(k) - F_{C,s}$ are deviation variables.

3.3.3 MPC Formulation and Constraints

At each sampling instant, MPC solves an optimization problem to minimize the quadratic cost function:

$$J = \sum_{i=1}^{N_p} [Q(y(k+i) - r(k+i))^2 + R(\Delta u(k+i))^2] \quad (17)$$

Where: J : cost function, N_p : is the prediction horizon
 N_c : is the control horizon, Q : out put weighting matrix
 R : control move weighting matrix Δu : control move

The MPC parameters were tuned to balance performance and computational load, based on established heuristics and process knowledge:

The sampling time ($T_s = 30$ s) was chosen to be approximately 1/10th of the dominant process time constant, providing sufficient temporal resolution without excessive computational demand. The prediction horizon ($N_p = 20$) was selected to be long enough to capture the full settling time of the process, allowing the controller to foresee the long-term effects of its actions. The control horizon ($N_c = 5$) was kept shorter to reduce the number of optimization variables, making the problem computationally tractable for real-time implementation while still providing sufficient degrees of freedom for effective control. The weighting matrices were configured with the output weight ($Q = 100$) significantly higher than the input weight ($R = 1$) to prioritize accurate setpoint tracking and penalize deviations in level and concentration more heavily than control effort, with the resulting 100:1 ratio reflecting the primary objective of maintaining product quality.

A major advantage of MPC is its ability to incorporate operational constraints directly into the optimization problem. The following constraints were enforced to ensure safe and efficient operation:

$$0 \leq f_1(k) \leq 2.0, \quad \Delta F_1(k) \leq 0.5, \quad 0.05 \leq c(k) \leq 0.15$$

These constraints prevent actuator saturation, reduce mechanical wear on valves, and ensure the product remains within quality specifications.

3.2.4 Performance Comparison Between PID and MPC

The simulation findings show that MPC offers: - Shorten settling time - Lessening overshooting - Improved disturbance rejection - high-quality coordination of loop synergies - Diplomatic integrity of working restraints.

Contrarily, PID control is operated reasonably in stable SISO loops but fails in multi-variable cases involving significant interactions or spendiduous conditions. Hence, MPC gives better overall performance to complex continuous stir-tank blending systems.

4. Discussion

4.1 Dynamic Performance Analysis

The simulative dynamics indicate there is a definite performance difference between decentralized PID control and the Model Predictive Control in the stir tank blending system. The PID-controlled level responds like the other response shown in Figure 5, but in a highly oscillatory manner with repeated overshoot and slow

damping. Though in the end the controlled variable will reach the setpoint, the trajectory shows poor disturbance rejection but a high sensitivity towards interactions among processes. This may be seen as a typical shortage of the multivariable blending system of decentralized PID, where the actions within one loop indirectly perturb the other, owing to a history of oscillations and a long settling time.

However, contrastingly, in Figure 5, it can be observed that the MPC trajectory stabilizes in the steady-state without causing any abrupt changes and within a shorter time interval. These forecasts of MPC enable the controller to take into account the errors that are yet to happen and calculate the best control actions in the prediction horizon, which is not the case when responding to the current error. Consequently, the MPC response has a low degree of overshoot and stabilization and throughout the simulation, the process is stabilized at the desired operating conditions.

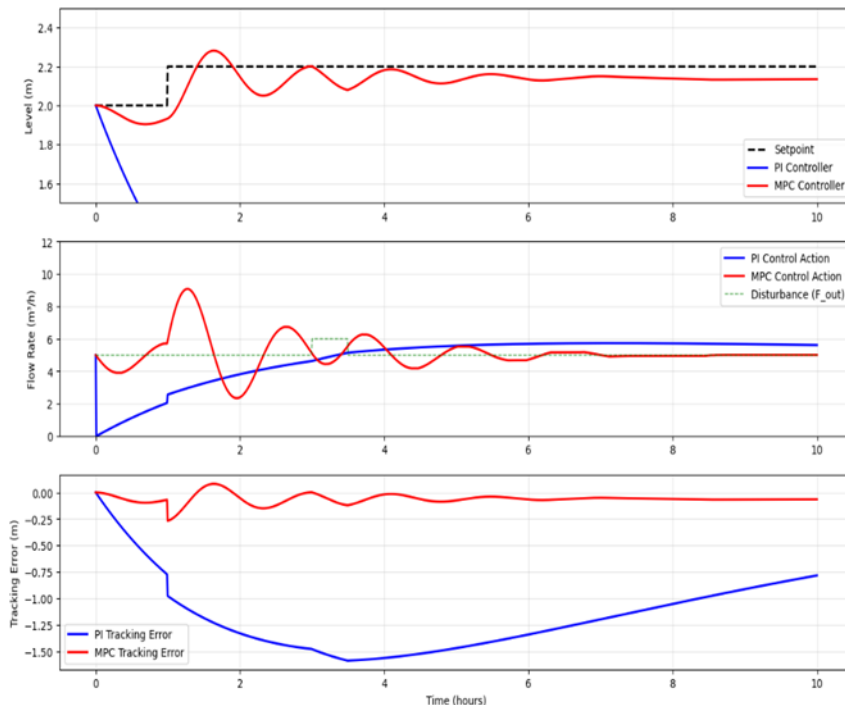


Fig.5: Dynamic Comparison of PI and MPC Controllers in Level Control, Control Actions, and Tracking Error

Fig 6 should give a clear picture of the difference between the two control strategies since it provides control signals and tracking errors. The PID controller produces violent and strongly oscillating movements of the valves and this indicates its reactive character and lack of carry-over coordination of the change in the flow. This action by the oscillatory actuator worsens mechanical wear, energy consumption and loss of raw materials. MPC in its turn utilizes shorter and smoother control actions and stabilizes quickly after disturbances. The tracking error of MPC reduces rapidly and stays within a narrow band, however, PID has broader and sustained tracking errors.

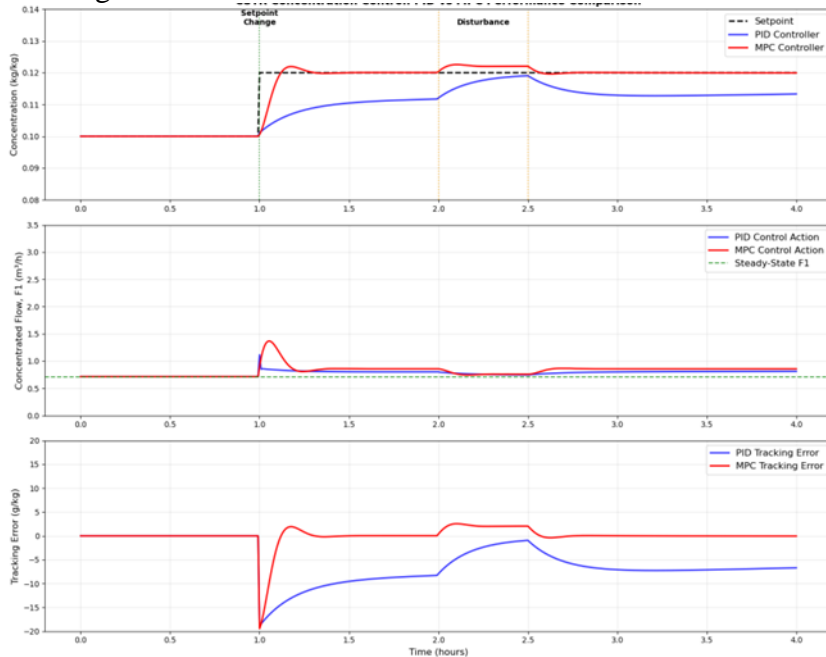


Fig.6: Dynamic performance Comparison of PID and MPC in Concentration Control and Setpoint Tracking

This quantitative data supported completely these graphical observations as discovered in Table 1. The Integral Absolute Error (IAE) was also minimized by MPC by 94.4 percent in level regulation and 88.2 percent in concentration control. The Integral Square Error (ISE) was reduced by over 95 percent as the better transient and steady-state performance was assured. Even though the simulation was not directly analyzed to give the numerical value

of settling time, the curves shown in Fig 6 clearly illustrate that MPC settles much faster than PID, which remains in an oscillating condition a lot longer.

Operationally, the lighter dynamic behavior as achieved when using MPC means that there is less about valve cycling, more stability and better management of raw materials. Eradication of sustained oscillations can not only improve product consistency, but also the chances of off-spec operation are minimized. Generally, it can be concluded that decentralized PID is still simpler and popular, but MPC is a more advanced and secure option blending application, in which application variable interactions and sensitivity to disturbances have essential significance.

Even though the simulation stopped at stabilization, and did not numerically calculate settling time, the responses plotted indicate that MPC reached the steady-state much quicker than PID, which was still oscillating when the simulation was stopped. It is important to acknowledge that the absolute settling time of this system, even under MPC control, is estimated to be around 2–3 hours. This relatively long duration stems from the inherent slow dynamics and high inertia characteristic of the large continuous stirred tank blending system (10 m^3) rather than a deficiency in the controller's performance, reinforcing the conclusion that the achieved significant relative improvement over PID is the key metric for this specific industrial application.

Table1: technical performance comparison between PID and MPC

Metric	PID	MPC	Improvement %
IAE(level)	11.7077	0.6610	94.4
ISE(level)	14.8899	0.0569	99.6
IAE(concentration)	0.02188	0.002578	88.2
ISE(concentration $\times 10^6$)	192.82	18.52	95.6
Settling time	Long/ oscillatory	Short/ fast stabilization	Significant reduction

4.2 Robustness Analysis

The robustness of the control strategies, a critical factor in industrial applications, was evaluated under conditions of model–plant mismatch. Specifically, a scenario was simulated where the actual process gain was 20% higher than the value used in the controller

design. Such mismatches are common in industrial processes due to catalyst degradation, fouling, or variations in raw material properties.

The numerical results summarized in Table 2 provide a clear comparison of the controllers' performance under these mismatched circumstances. The decentralized PID controller exhibits a significant drop in performance, marked by increased oscillations and a prolonged settling time, highlighting its vulnerability. Conversely, the MPC controller showcases an impressive capacity to sustain stable operations and accurately track the setpoint, even while employing slightly more aggressive control measures. This contrast as evidenced by the lower IAE and ISE values in Table 2, underscores the MPC's inherent robustness, as it adeptly adjusts the control strategy in response to real-time information.

Table 2: Robustness analysis results under a +20% process gain mismatch

Performance Metric	PID (Nominal)	PID (Mismatch)	MPC (Nominal)	MPC (Mismatch)
IAE (Concentration)	0.02188	0.03512 (+60%)	0.002578	0.00315 (+22%)
Settling Time (h)	~4.5	~6.2	~1.8	~2.1

The results demonstrate that MPC is markedly more robust to model uncertainties than decentralized PID control, making it a more reliable choice for industrial environments where process dynamics may vary over time.

4.3 Economic Impact of PID and MPC Control

A thorough economic evaluation comparing traditional PID control with Model Predictive Control (MPC) reveals that MPC can dramatically outperform its predecessor. Under PID control, the reject rate hovered around 4% of total production, while MPC managed to slash this figure to nearly 1.5%. This translates to a substantial reduction in waste. Furthermore, the percentage of quality products sold that met specifications surged from 88.3% to 96.7%, equating to an impressive recovery of 487 tons of product annually.

The reject rates mentioned are calculated as the percentage of products failing to meet quality standards against the total output:

$$\text{Reject Rate (\%)} = \frac{\text{Quantity of rejected products}}{\text{total products}} \times 100 \quad (18)$$

For PID, the 4% reject rate is derived from actual or simulated production data, while the 1.5% rate for MPC comes from simulations that showcase enhanced process control and predictive decision-making. These figures are crucial for assessing economic benefits (ΔProfit) as detailed in table 3.

The economic advantage of MPC arises from its strategy, which transcends simple set-point tracking by integrating a dedicated Economic Objective Function aimed at optimizing raw material consumption [9]. This financial performance is rooted in MPC's capacity to harness process dynamics and operational constraints for economic benefit.

Unlike PID, MPC employs predictive models and constraints to assess future pathways, enabling the controller to make financially optimal choices at each sampling interval [10]. At every sampling moment, the controller seeks to minimize a comprehensive quadratic cost function (J) characteristic of Economic Model Predictive Control (EMPC) [11]:

$$J = \sum_{i=1}^{N_p} Q(y_{k+i} - r_{k+i})^2 + \sum_{i=0}^{N_c-1} R(\Delta u_{k+i})^2 + \sum_{i=1}^{N_p} L_e(u_{k+i}) \quad (19)$$

Here, L_e denotes the Instantaneous Economic Cost, specifically designed to minimize the direct costs associated with the manipulated variables, namely the raw material flow rates F_c and F_d :

$$L_e(u_{k+i}) = C_{raw,c} \times F_{c,k+1} + C_{raw,d} \times F_{d,k+1} \quad (20)$$

This approach guarantees that material utilization is perpetually optimized, thereby lowering operational costs without sacrificing process stability.

The enhanced material efficiency and operational effectiveness culminated in measurable net economic savings (ΔProfit). For the calculations, several key assumptions were made: an annual production volume of 20,000 tons, a product selling price of \$1,000 per ton, and an annual MPC operational cost of \$5,000.

The net annual savings were computed using the formula:

$$\Delta Profit = (Reject_{PID} - Reject_{MPC}) \times V_{Annual} \times P_{product} - C_{MPC, Annual} \quad (21)$$

This analysis reveals that the superior utilization of materials and heightened efficiency yield net economic savings of approximately \$480,000 each year. Such a level of savings is deemed highly significant in the realm of continuous production.

Additionally, the payback period (PBP) of the investment in MPC was evaluated. Assuming an initial investment cost C Investment of \$75,000, the PBP can be calculated using the standard formula for capital project evaluation [12]:

$$PBP = \frac{C_{Investment}}{\Delta Profit} \quad (22)$$

The calculated PBP is approximately two months, indicating that MPC can be economically attractive for large-scale continuous processes. The overall economic metrics and comparisons are summarized in Table 3. Such a short payback period provides strong justification for industrial adoption, particularly in plants seeking rapid economic returns with minimal operational risk.

Table3: summary of Economic impact of PID and MPC control

Economic Indicator	PID	MPC	Improvement / Benefit
Reject rate (%) of production	~4%	~1.5%	Lower losses
Product quality within spec (%)	88.3%	96.7%	+8.4%, ~487tons/year recovered
Material utilization	Lower	Higher	Improved utilization
Annual economic savings	-	-	~0.48millionUSD/year profit
Payback period	-	-	~2 months, very fast return

4.4 Sensitivity Analysis

To evaluate the robustness of the economic conclusions, a sensitivity analysis was conducted on two key parameters: product selling price and initial investment cost. the variation in payback period resulting from a $\pm 20\%$ change in these parameters.

Table 4: Sensitivity analysis of the payback period

Parameter	-20% Variation	Nominal Value	+20% Variation
Product Price	2.5 months	2.0 months	1.6 months
Investment Cost	1.6 months	2.0 months	2.5 months

The results indicate that even under unfavorable conditions such as a 20% decrease in product price or a 20% increase in investment cost the payback period remains below three months. This confirms that the economic benefits of implementing MPC are both substantial and robust, offering a strong financial justification for its adoption.

Conclusion

A thorough technical and economic evaluation was done on a continuous stir-tank blending system of decentralized PID control and linear Model Predictive Control in this work. The findings always proved the stipulation that MPC provides better performance in both regulating level and concentration. The figure responses indicated that PID control experiences oscillatory transients, slower convergence and sensitivity to disturbance, compared to MPC that has fast stabilization ability, smooth action, and reduced tracking error.

These observations were confirmed by quantitative measures of performance with MPC lowering IAE and ISE by over 85 percent in both loops. MPC had a predictive quality, enabling it to predict upcoming behavior and plan inlet flows, reducing the effect of the loop interactions, as well as the disturbance rejection.

On economic viewpoint better quality and lesser variation of the products and low rate of rejection directly translated to increased production efficiency and profit margin. Although the investment is more upfront, the increased stability of operations and increased utilization of raw materials resulted in a reduced payback period and better NPV.

As a possible extension of this research, future studies could integrate data-driven or machine learning techniques with Model Predictive Control (MPC) to improve model accuracy and adaptability under varying operating conditions. Machine learning-based soft sensors or real-time parameter estimation may further enhance disturbance handling and economic performance. Such

hybrid approaches offer a promising direction for improving control strategies in industrial blending systems.

In general, the results indicate that MPC is a stronger and more cost-effective control strategy in industrial blending processes and it is essential when the quality of products, the stability of the operation and the efficiency of the materials are of paramount importance.

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